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What Can the Teacher of Mathematics Do About Vocational Guidance?

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What can teachers of mathematics do about vocational guidance? First of all, whether intentionally or not, every teacher of mathematics plays an important part in vocational guidance, for today, with young men throughout the country taking examinations to enter some branch of the army or navy their success is in a very large measure determined by their ability to pass the test in mathematics. Some of them due, in part at least, to thorough training pass easily, while far too many have difficulty or utterly fail. It is therefore quite probable that teachers who saw to it that their pupils actually learned arithmetic and algebra did the finest possible work in vocational guidance although they may never have heard of the term. At any rate, at the present time, the first duty of teachers of mathematics is to make certain that students really know fundamentals. Unless this is first done, talk of vocational guidance in mathematics is worse than useless.

Next, the teacher can do his part to see that the curriculum is such that the pupil will be trained so that he can enter vocations for which he is naturally fitted and which are open to him. Here, we meet the problem which has been under discussion for the last few years, how shall we provide for the group which needs more work in arithmetic and at the same time not hinder the progress of those who can profit by studying as much mathematics as possible. As teachers of mathematics we know only too well that there are many who have not learned arithmetic when they enter high school. It is superfluous to show that knowledge of arithmetic is necessary in most walks of life, but in the light of present conditions it is doubly important. Even in the infantry, the least technical branch of the army, the subcommittee of the war preparedness committee pointed out the need of enlisted men to use arithmetic. Skilled workers in industry use arithmetic and algebra. The same is true of nurses. It is obvious that arithmetic must be given in high school for those needing it. On the other hand, adequate provision must be made that those who already know arithmetic have the opportunity to get as complete a course as possible in secondary mathematics.

Professor William L. Hart in a report of the war preparedness committee says,

> I recommend that in the junior and senior high school every boy and girl of sufficient mathematical aptitude should be urged by high school advisers to take as much mathematics as possible, through the stage of trigonometry and some solid geometry as a national service.

Many of you no doubt read in the MATHEMATICS TEACHER of February, 1942, the letter written by Captain F. U. Lake of the Bureau of Navigation at the direction of Admiral Nimitz. Here is a part of that letter.

> A carefully prepared selective examination was given to 4,200 entering freshmen at twenty-seven of the leading universities and colleges of the United States Sixty-eight per cent of the men taking the examination were unable to pass the arithmetical reasoning test. Sixty-two per cent failed the whole test, which included also arithmetical combinations, vocabulary and spatial relations. The majority of the failures were not merely borderline, but were far below passing grade. Of the 4,200 entering freshmen who wished to enter the Naval Reserve Corps, only ten per cent had already taken elementary trigonometry in the high schools from which they had graduated. Only twenty-three per cent had taken more than one and a half years of mathematics in high school.

The letter states that experience indicates that seventy-five per cent of the failures in navigation must be attributed to the lack of adequate knowledge of mathematics.

As teachers of mathematics, we are naturally blamed for this and the question arises, what can we do. First, we can do all in our part to see that pupils learn what we teach. Next, we can see that they have an opportunity to take mathematics at least through plane trigonometry. It is no longer a question of those going to work and the college preparatory group. A great many of our boys and some girls are going to want to take mathematics in high school because they will have little opportunity later.

In the past, the work in mathematics has been severely criticized for not functioning in life. Probably no item will do more to improve the teaching of mathematics in this respect than a study of its use in various vocations. At once we are made to realize that we must continue to stress fundamental principles and teach mechanical skills but that we must also show how these skills are used. An article on nursing for example suggests problems on the strength of solutions as an application of proportion. For some of us, it is difficult to get real applications.

Where does one find information on mathematics and vocational guidance? Since now the most important field is in defense, the two reports of the war preparedness committee are very useful. The committee is one from the American Mathematical Society and the Mathematical Association of America. A year ago today, February 21, 1941 Professor Marston Morse of Princeton University delivered before the National Council at Atlantic City an address on the work of that committee. This with portions of an address of Professor William L. Hart of the same committee was published in THE MATHEMATICS TEACHER for May 1941 and later republished in the *American Mathematical Monthly*. Recently, reprints could be obtained in units of twenty-five from Professor W. D. Cairns, Oberlin, Ohio. The report is very comprehensive. The account of the subcommittee in research will interest your more advanced pupils. It discusses briefly some aeronautical problems with mathematical solutions, the work in ballistics at the government proving grounds, the computation done by means of the huge machine the Bush Analyser, the use of mathematics in cryptanalysis.

The part of the report that is especially valuable to high school teachers is the detailed account of the mathematics used in various branches of the service and in non-military defense. This is also fully discussed in the later report in THE MATHEMATICS TEACHER for November 1941. Throughout, the point is stressed that almost every branch including skilled work in industry would have been benefited if the men had studied high school mathematics through trigonometry.

Aside from the military service, there are certain callings which particularly require mathematical training. Of these engineering is outstanding. In July, 1941, Nation's Business said, "Engineering schools have from three to five employ-

ment offers for every graduate." The New York *Times*, June 8, 1941 said that the demand for graduate engineers at Columbia had increased four hundred per cent. Positions were open to women especially those with chemistry, physics and Spanish. On March 2, 1942 the University of California opened a twelve weeks course for women to train for positions in drafting and technical shops of vital industries. The prerequisites are one or more years of college, including physics and mathematics. It is a special course given by the government and the university.

Recently, a monograph by Thornton C. Fry, mathematical director of the Bell Telephone Laboratories was published. He estimates the total number of mathematicians in industry as one hundred and fifty. He is speaking of the expert in research. He says that nowhere in America is there a college to train industrial mathematicians. Although there must be a background of higher mathematics, according to him algebra, trigonometry and the elements of calculus are the most productive in modern research.

He writes, "The single side band system of carrier transmission was a mathematical invention. It virtually doubled the number of long distance calls that could be handled simultaneously over a given line. Yet the only mathematics involved was a single trigonometric equation, the formula for the sine of the sum of two angles."

Another important line of work in applied mathematics is that of the statistician. Professor Hart in the war preparedness report said, "For these non-military activities, as many women as possible should be trained through substantial high school mathematics; a more select group through the stage of elementary statistics to create a reservoir of computers for government and industry." This has already begun. Many are employed by the federal government, the states and private industry.

A glance at the first page of the syllabus for the examination of the Actuarial Society of America shows a three hour examination in algebra, three hours for calculus, three for the theory of probability and three for compound interest and annuities.

Accounting is another opening for the mathematically inclined. Among the qualifications listed by the American Institute of Accountants are: ability to assemble and to base ideas on unrelated facts; to interpret their significance and to express conclusions in correct and accurate language; a sense of perspective which will not be distorted by proximity to minor details; great respect for mathematical accuracy. Higher mathematics, it states, is seldom used in the ordinary practice of accounting although elementary algebra is valuable.

A worthwhile service might be a survey of the vocations suitable for those interested in mathematics. These include some requiring comparatively little mathematical training like that of the bookkeeper, calculating machine operator, estimator or draftsman as well as those requiring a great deal of advanced work like that of the research mathematician or physicist. To a high school teacher reading along this line, two points stand out. First, the people going into defense occupations should have training through plane and spherical trigonometry if they are capable of learning it, and all of these must know arithmetic. Secondly, those going into scientific study or engineering need a thorough foundation in algebra. As teachers, we shall find that the more we know about the vocations which our students may possibly enter, the better teaching we shall do. So the question perhaps should be, not what can the teacher of mathematics do about vocational guidance but what can vocational guidance do for the teacher.

The Responsibility of the Mathematics Teacher in Curriculum Building

By Judson W. Foust
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For some time two important activities have been attracting the attention of mathematics teachers. The first is the criticism of mathematics, both constructive and destructive. This criticism has come from within the ranks of mathematicians as well as from without. It has been concerned with the purpose, content, methods, and results of mathematics at various levels. The second activity has been concerned with a reorganization of the whole school curriculum including among other considerations a careful evaluation of the effectiveness, in various subjects, of innovations in sequence, time allotments, grade placement, and requirements for graduation or college entrance.

As a part of all this many experiments have been set up, discussions are being held, and conclusions will be made that will result in a reorganized curriculum. A fair question to consider, and an important one, it seems to me, is the following—who will make these curricular decisions? As a partial answer it is here proposed that at least where mathematics is concerned decisions should be made with considerable cooperation from those having not only a natural and deep interest in mathematics but who also from training and experience have considerable insight into the nature and contribution of mathematics.

Curriculum building is concerned with what is to be taught, to whom, and when. Viewing this in a practical way I recall a remark of a history teacher after reading from the constitution the manner in which certain officials are elected. "This," he said, "is the way the constitution says they are elected, but now I will tell you how they really do get elected." My remarks will likewise merely consider some responsibility mathematics teachers have toward certain factors that influence the curriculum and the way in which the curriculum is actually administered where guidance plays a part in the election of any subject.

The need to bestir ourselves is not that the situation is changing unfavorably in any detail. In fact, within the past two years there has been apparent a definite demand from many quarters for increased mathematical training. The need comes from the desire to clarify and improve the situation confused by too many uncritical unsubstantiated, misleading half-truths and inferences. The need also arises because of the inconsistency created by those demanding the application of certain educational principles in one area and ignoring or denying their usefulness in other fields. A few specific illustrations may be worth considering.

A few weeks ago a prominent speaker in quoting the results of an experiment in learning mathematics in college by a group lacking certain high school units left the impression that mathematics could well be postponed until college and further that college mathematics could be studied with equal success by those who had or did not have high school mathematics.

A member of a State Department of Public Instruction recently stated in a guidance meeting with high school students in response to questions on the part of those interested in the many recent articles on the need for mathematical training that mathematics was a good subject but it should be taken in college.

Two members of the same Department of Education within a week of each other made these statements. Said one, "There is no need to study arithmetic beyond the seventh grade." Said the other "There

should be no arithmetic taught until grade eight."

Such statements are both confusing and misleading. Certain schools have interpreted the relaxing of graduation standards and college entrance requirements, through the removal of algebra and geometry from the required list, as an indication that it is now admitted that these subjects are of little value. In certain instances the next step has been to not even offer these subjects. In less extreme situations the student has been permitted to assume that the minimum is a satisfactory maximum. Many of the ardent advocates of individual differences who sought to have the requirements relaxed to care for the weaker students are not now as strong in their championing of the principle of individual differences to the end that the capable and interested student not only be given the opportunity to study mathematics further but that he at least be encouraged to do so. We should ask them why and insist on a consistent application of the principle of individual differences.

There has likewise been much talk in recent years of *adapting the curriculum, the program of studies, and the subject matter to the interests of the child*. Discussing this in their recent book Butler and Wren[1] write,

> This phrase carries with it the implication that the interests of the child constitute a safe and sufficient guide for planning the experiences through which the child is to be educated. If, however, children are to be brought to intellectual and emotional maturity, their experiences must be planned and guided by something more than their own temporary and evanescent interests. Their interests need not only to be guided but to be *awakened* and *expanded* through appropriate experiences. . . . Interest and experience interact. Interest is not generally speaking spontaneous and inherent; it is excited through experience. On the other hand, interest stimulates activity in experiences. Thus interest is a product of experience as well as a stimulus to experience.

The Joint Commission[2] emphasized this viewpoint when it stated,

> For it must be remembered . . . that the school has an obligation to create capacities of one kind or another and should explain to pupils the advantages which may result from them, though it recognizes that in many cases the capacities will not all be employed.

Coupled with the responsibility which the foregoing implies is another factor of equal importance. The three new national reports in the field of mathematics are all committed to a program of *systematic* continuous training in mathematics. This was emphasized last year at Atlantic City by Mr. William Betz. It is re-emphasized by Butler and Wren[3] in the following words.

> The values of mathematics are not, as a rule, of such a nature as to be attained informally or incidentally. Their acquisition generally requires serious and sustained application to subject matter characterized by sequential continuity and cumulative organization. . . . However, the demands which mathematics makes upon the persistent and serious application of those who study it cause it to be avoided, under the free elective system, by many students who could profit largely from it but who perfer rather to fill their programs with less arduous and less substantial courses. Unless such students are wisely advised and encouraged to continue their training in mathematics and in other substantial subjects as far as their capacities will permit them to profit from these studies, we need not be surprised to see the enrollments continue to decline. Education is too large and too serious a business to be dominated by a superficial philosophy of evanescent interests and transitory values. The minimum must not be allowed to become the norm. Those whose responsibility it is to counsel with secondary school students should help them see beyond the immediate present; to point out to them potential values which at the moment are perhaps not obvious; and to give them a preview of those insights, appreciations, higher satisfactions, and higher instrumental values that lie 'beyond the threshold' of mathematical study.

If teachers of mathematics do not feel a responsibility to concern themselves with these important items, we certainly cannot expect others to do so. Rather in light of these suggestions we should propose that

[1] *The Teaching of Secondary Mathematics*, pg. 69, McGraw-Hill Book Co., 1941.

[2] Joint Commission of the Mathematical Association of America, Inc., and the National Council of Teachers of Mathematics. *The Place of Mathematics in Secondary Education*, pg. 207, Fifteenth Yearbook of the National Council of Teachers of Mathematics, 1940.

[3] *Op. cit.*, pg. 71.

those who believe in the theory of interests should be taken at their word and insist on a wide variety of interests and experiences and opportunity for each pupil to develop to the limit of his capacity. This must also include, let us remember to remind them, a wide variety of experiences in mathematics. It must be sufficiently inclusive as to material and pupils to insure the awakening of the interest and the development of the capacities of all that have ability in this direction. The demands of a scientific, machine age are too great to allow these gifted ones to pass by. In the recent words of W. D. Reeve,

> . . . we must open up mathematics, so to speak, so as to give each child a chance to see what the subject means, what his likes and dislikes are, and in what direction his future interests lie. I feel perfectly sure that we must teach not so much mathematics but more about mathematics.

It is not enough for a student to be told no mathematics is required for graduation from high school or no mathematics is required to enter college. The other half of the story needs to be told. Consider the added information given by following the statement that one can enter the University of Wisconsin without mathematics by this statement from their Entrance Requirements.

> Restricted admission opens to the student such Colleges, Courses, and fields of specialization as do not require high school mathematics as background. It does not give admission to the College of Agriculture or the College of Engineering or the Course in Chemistry and does not permit the student to major or specialize in chemistry, commerce, economics, mathematics, pharmacy, pre-medicine, philosophy, political science, psychology or sociology, or in any other of the natural sciences including physical geography, and geology, or to graduate from the School of Education with a major or minor in any of these fields.

Any careful consideration of the suggestions just discussed reveals the fact that every mathematics teacher should be deeply concerned with certain aspects of curriculum building. It is not a question of whether we as teachers of mathematics will have an opportunity to contribute to curricular decisions but rather to what extent will we recognize the opportunities, feel the responsibility, and take the time and effort needed to make a real and lasting contribution. Disinterest in general educational problems is no excuse, for whether he likes it or not, the mathematics teacher must realize that he is part of the total educational program and as such must make his contribution to that total process.

We must not turn so much of our energy to internal improvements in the teaching of mathematics that we neglect the task of setting that field in its proper place with respect to the total educational picture. Unfortunately it is not true that mathematics will grow in favor in direct proportion to the improvement in its teaching. Many changes in the curriculum are made by individuals or pressure groups who are by no means personally disinterested in the outcomes. General educators and school administrators could use and generally would welcome the considered opinion of subject matter groups that show an interest in and insight into the problem. We must recognize that there is no detached, unprejudiced, uninterested, all-wise builder of curricula. Rather, any curriculum emerges as a compromise, a fusing, a temporary balance of the values and points of view concerning the contributions of various subjects all considered together with respect to some educational philosophy or goal. It is my thesis that the mathematics teacher has the responsibility for presenting the case of mathematics.

In so doing advantages will accrue to all concerned. It will require the mathematics teacher to become familiar with what is being done in the general field of curriculum revision, to get a deeper insight into and appreciation of the total educational plan, and to re-evaluate and clarify in his mind the contribution of mathematics. In the second place any fresh, objective consideration of the claims of mathematics should lead to a knowledge of the shortcomings of mathematics, a knowledge of the extent to which the criticisms against

mathematics are valid, and a knowledge of the limitations of the contribution of any single subject. Lastly it should stimulate experimentation with curricular patterns on the part of those with adequate training and experience in order to suggest the next steps in mathematics instruction. All of these activities should make us better teachers of mathematics.

Perhaps of greater importance will be the contributions our participation will make to the general problem of curriculum building. I mention only three.

1. There should result a better understanding of the purpose of mathematics—what mathematics really is. This lack of understanding on the part of the student and the general public as to the nature of mathematics I consider one of the greatest handicaps today to a wider acceptance of mathematical study. Is there any good reason why everyone shouldn't be let in on the secret? In fact, shouldn't we make it our first and final goal in teaching mathematics to have students understand what it is all about? To put it very briefly mathematics is concerned with problem solving as an end. Its method is the study of relationships —quantitative relationships, spatial relationships, logical relationships. Two basic characteristics of mathematics are that it is arbitrary and abstract. This is the outline of the story that must be filled in and expanded. Until this outline is understood mathematics will not be understood. It is the mathematics teacher who must clarify it again and ever again.
2. Participation on the part of mathematics teachers in curriculum building would furnish a partial balance for undue pressure against mathematics through an active interest and first hand information on various aspects of the problems.
3. There is much to be gained by all through a cooperative attempt at solving a common problem. We can not only contribute our first hand knowledge of our own field but can profit from the detached and varied points of view of those looking on from another angle and at a vantage point slightly removed. In fact we cannot hope to teach mathematics only in the mathematics classroom nor English only in the English classroom. Our teaching must in many ways overlap or at least be reenforced. To do this effectively we must understand and appreciate to some extent various fields and various fields must likewise understand and appreciate mathematics. This can best be secured through working together on some common problems including curriculum building.

The Michigan Department of Public Instruction has for several years conducted a State Conference on Curriculum and Guidance. As chairman for the last three years of a section considering the topic, "Helping Students Grow in Their Ability to Think," I have found a deep interest in this basic problem on the part of teachers in various subject matter fields. By considering a common problem there has resulted a better understanding of and a new respect for various subjects. The approach is positive in that we are not criticizing this subject or that but exploring all possible ways of reaching a worthwhile goal—in this instance the contribution of each subject to the goal of helping students grow in their ability to think. I know that I feel more keenly the need for aid on this problem from other fields and I trust they are more informed and interested in what mathematics can contribute.

We will not all be interested in the same aspects of curriculum building. The mathematics teacher does, however, have a responsibility in this area. It is considerable. Active participation on our part in curriculum building in whatever way we find an opportunity and interest should prove most helpful to the whole school program.

School and Community Look at the Content of Consumer Mathematics

By Walter A. Wittich
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In the Madison Public Schools, we have a situation where curriculum problems and course of study evaluation and revision are carried on as a cooperative effort by teachers, principals, and supervisors. Early during the current year, a committee of junior high school mathematics teachers, heads of mathematics departments, an administrator and a supervisor met to suggest a means by which they could evaluate and make more meaningful to the junior high school pupil who was not going into college preparatory work, the study of arithmetic and mathematics as he would meet it on graduation from high school.

After conducting a standardized test survey to determine status and after teachers had met to give voice to their thinking on what should be included in the type of work followed by young people who were not headed in the direction of college but rather who would go directly into local industry and distributive mercantile businesses on graduation, it was their suggestion that a survey of Madison union apprentice educational advisors, personnel officers, and business men be made in an attempt to determine what mathematics skills are expected of high school graduates whom they hire and will continue to hire. In addition, it was suggested that these men be interviewed for the purpose of evaluating the mathematics training held by persons who entered their employment during recent years.

Interviews were accomplished with representatives of twenty-two of the aforementioned officers of local industry, mercantile establishments, and unions. So as to give uniformity to the interviews, questions were asked concerning what arithmetic and mathematics knowledges and skills people entering into their threshold jobs would have to have in order to carry on satisfactorily the expected work. Questions which designated skills in the areas of addition, subtraction, multiplication, and division of whole numbers and decimals were asked and last, the business representative was asked to express himself on the type of mathematics training he thought would best serve the young person who graduated from high school and entered not only into business life but into the social life of the community.

The following is an item by item tabulation of the responses made by the twenty-two business organizations, labor unions, educational advisors, and personnel managers interviewed: (The figure at the left indicates the number of times the item was mentioned as being important.)

8 linear measure: ½ ft., ¼ ft., ⅛ ft.
7 quick and accurate mental addition, subtraction, multiplication, and division, as in making change, making multiple sales of similar items, estimating costs, etc.
7 linear measure: ½ yd., ¼ yd., ⅛ yd., ¼ in.
6 understanding carrying charges, instalment buying, costs of small loans, automobile purchase, etc.
6 linear measure: ½ in., ⅛ in., 1/16 in.
5 ability to make change up to $10.
5 ability to compute set mark-ups as ordered by store managers to departments.
5 ability to estimate amounts of materials: paint, wood, sand, concrete, etc., to be used on a job of specified size.
5 ability to keep accurate records of hours worked, materials used, etc.
5 ability to understand and use measures of liquid and dry quantities such as are involved in the number of servings per gallon of ice cream, etc.
4 ability to compute unit prices, such as are found in grocery stores: cost of one unit if prices are 3 for 25¢, 5 for 29¢, 3 for 20¢, etc.
4 ability to interpret charts and tables and other pictorial statistics.
3 ability to take inventories in which items are added and subtracted.
3 ability to determine value received in canned goods and package groceries when comparative weights are considered.
3 ability to write numbers legibly.
2 ability to handle square measure in terms of feet and yards.

2 ability to handle cubic measure in terms of feet and yards.
2 ability to interpret blueprints, building plans, landscaping layouts, etc.
1 know the fractional parts of the year and month.
1 estimate the reasonableness of a bill, the reasonableness of a meter charge.
1 estimate the comparative efficiency of various electrical appliances.
1 ability to read meters.
1 ability to lay out 30, 60, and 45 degree angles, as in carpentry.
1 ability to reduce fractions to common denominators and perform the four fundamental processes.

The response of the same lay persons to the question: "How may the school better equip its graduates so that they may take a more intelligent part in the social life of the community?" revealed, perhaps, the most interesting information of the survey. Such responses as these were forthcoming:

"Perhaps, one of the greatest needs of our young people is a knowledge of interest, of instalment-buying plans, time-payment plans, and other interest arrangements on deferred-payment purchases. Plans at present in existence vary tremendously to the advantage or to the disadvantage of the consumer, and it becomes a real problem both to the seller and to the buyer to choose between the many time-payment arrangements and plans which are continually advertised in the local newspaper or over the radio both by the firm of long-standing reputation and by the 'fly-by-night' operator who offers attractive plans which, in reality, contain pitfalls in the form of technical provisions which, on the surface, appear innocent but which underneath contain hidden arrangements which may cost the unwary buyer a great deal of money in addition to mental anguish."

"It certainly must be the school's responsibility to acquaint all of its students with understandings of property ownership, property insurance, property taxes, methods of financing homes, and home purchase plans. If the school does not acquaint the young people with the existence and operation of F.H.A., insurance company, bank, building and loan, and private loan agencies' methods of financing home ownership, then they will go out into the community many times not having any valid basis for the comparison of these and other plans. In addition, it seems to me, the schools could do more to teach an understanding of life insurance plans, understandings of inheritance tax, income tax, and all of the other means by which organized government gathers revenue for the purpose of providing services."

"It seems to me that we try to put all of our children through the same mill not only in mathematics but in other of our school subjects. In mathematics, it's my opinion that the school doesn't think it has done a good job until it tries to give all of its students enough 'math' training so that they can go out and be engineers, or until they give all physics students enough physics so that they can go out and become expert research people. Why can't you in the schools teach more of the kind of mathematics that my daughter will use every time that she walks into a grocery store or department store, or that my son will need when he attempts to understand what insurance agents, automobile finance men, or real estate agents try to sell him on their plans. It seems to be too much of the old story of teaching our children at what temperature water boils at the top of Pike's Peak or down in Death Valley and forgetting to show them how to boil water for tea."

"In general, the arithmetic skills that we ask for are very simple, but it's most important that our people have the ability to make the simple computations quickly and accurately. For example, many of our people have great difficulty in making change. If a customer hands them a 50¢ piece and two pennies in payment for a 27¢ purchase, it is with great difficulty that the girls make change, which, of course, is done by taking the two pennies from the price of the purchase and continuing from there."

Naturally, when the composite of the survey was turned over to the committee for further consideration, it was with the recommendation that it represented one criterion or one viewpoint, but certainly a very valid one. Later, as the committee met to consider the type of a textbook to adopt for use by the children who were going directly from high school into the community, they used the item analysis of needs indicated by employers as one very important criterion of selection. While no textbook could possibly lend itself totally to the need of a specific community, the committee finally decided upon materials which came very close to supplying the type of understandings which the community felt its graduating high school students should possess. At those places where text materials did not

offer the understandings asked for, certain interested committee members undertook to supplement the text materials with units of work which they themselves created in an effort to offer students specific understandings not available in formal textual material. There is no need to mention again the value of conducting such a community survey. In every case, full cooperation was obtained, and it is our feeling that it is with every enthusiasm that lay persons in the community worked with school authorities in trying to arrive at some more valid and useful course of study content.

Air Navigation

By HALE PICKETT

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AS A TEACHER of mathematics, I have been interested in the opposition to mathematics as a required subject in the high school and college.

Although the critics have been severe, I have not lost faith, since I know that mathematics will always be needed for a scientific interpretation of the complex problems of life. Recent trends have been indicative of my faith in mathematics. Just as soon as our National Defense Program started, we found employers asking for men and women who have had training in mathematics. In line with these calls for mathematics, I am especially interested in the training of student pilots in our colleges. The recent request for Pre-Flight courses in our high schools by the Armed Forces should stimulate us to change our curricula to present needs. In other words, we cannot conduct our schools as usual. The rapid expansion of our Armed Forces demand leadership, and recent statistics indicate that our high schools and colleges are potential reservoirs of this leadership. It is our duty to accept this challenge and prepare our pupils and students for leadership. In order that we may be more efficient in our efforts to develop pilots, I suggest that those who are interested secure copies of "Pre-Flight Aptidute Test" from the Supt. of Documents, Washington, D. C., thus using some guidance in selecting prospective student pilots.

It is not expected that we teach anyone actually to fly, but a candidate seeking a certificate as a pilot must pass rigid academic examinations based upon his knowledge of the theory of flight, which involves physics, physical geography and mathematics. I am suggesting that we give our boys the fundamentals of Practical Air Navigation to help them pass such academic tests. Secondly, a properly organized course in aeronautics involves practical applications of physics, physical geography and mathematics. Thus, we are given an opportunity to make those sciences more interesting and meaningful to our students. In fact, this topic offers an ideal opportunity for a progressive educational program of teaching.

The following topics in physics: parallelogram of forces, equilibrium, magnetic variation, horsepower, centrifugal force, drag, and velocity have practical applications in this course. As for physical geography; latitude, longitude, distance, time and their interrelations, climatic conditions, construction, reading and interpretation of maps and charts; play an important part in navigation. Mathematics is needed for solutions of the above suggested problems. Graphs, scale drawings, and trigonometry are necessary for the successful navigation of an aircraft.

We are entering, or we have already entered *THE AIR AGE*. Billy Mitchell has been vindicated, his prophecy of the dominance of aircraft over the battleship has been demonstrated by the sinking of superdreadnoughts. War or no war; the airplane is becoming more efficient and also more popular each year.

Only a few schools have offered courses in navigation for teachers, therefore, we do not feel adequately prepared to teach such courses. However, in lieu of the times, an alert, energetic teacher need not wait for such preparatory courses, if he has had training in physics, physical geography and mathematics.

As a high school text, I am recommending, *Elements of Aeronautics*, by Pope and Otis, published by the World-Book Co.

As a student pilot's course in college, I am recommending the following: *Practical Air Navigation*, by Thoburn Lyon, a Civil

Aeronautics bulletin #24 published by The Civil Aeronautics Administration, Washington, D. C. for $1.00. A more advanced text is *Air Navigation*, by P. H. Weems, published by McGraw & Hill, which gives in detail the Weems System of Navigation; price, $5.00. This system is recognized as a standard in navigation.

In addition, the following equipment is necessary:

I. Charts:

1. Sectional charts, there are 87 sheets for the U. S. at a scale of 1:500,000 or about 8 miles to the inch.
2. Regional charts, there are 17 for the U. S. at a scale of 1:1,000,000 or about 16 miles to an inch.
3. Radio direction-finding charts of the U. S. there are six (6) at a scale of 1:2,000,000 or about 32 miles to an inch.
4. An Aeronautical planning chart of the U. S. at a scale of 1:5,000,000 or about 80 miles to an inch.
5. Great Circle chart of the U. S. at a scale of 1:5,000,000 or about 80 miles to an inch.
6. A magnetic chart of the U. S. showing lines of equal magnetic variation at a scale of 1:7,500,000 or about 115 miles to an inch.

All the above are printed by the U. S. Coast and Geodetic Survey and may be purchased for about 40¢ each.

II. A compass course protractor; purchasable from Air Associates, Garden City, N. Y. for 50¢.

III. An air computer is included in the above Bulletin #24 (However, a Dalton Computer may be purchased from Weems System of Navigation, of Annapolis, Md. for about $40.00).

IV. Parallel rulers and spacing dividers are helpful but not necessary.

The essentials for navigation have been reduced to the five following instruments: watch, altimeter, compass, air-speed indicator and drift sight.

Therefore, a knowledge of their construction and use is essential. In the early days of aviation, a pilot followed railroads, rivers, roads and land marks. If he encountered bad weather and poor visibility it was just too bad. To-day there is no guess work in the navigation of a plane. It is now a scientific procedure.

Aerial navigation is the science of piloting a plane from one point to another on the earth's surface and establishing its position at any time.

There are four methods of air navigation, namely: pilotage, dead reckoning, radio navigation, and celestial navigation.

I. Piloting is directing a plane with respect to visible landmarks.

II. Dead reckoning is the method of determining the geographic position of an airplane by distance and direction from a known position. In other words, it involves the solution of the triangle of velocities.

III. Radio navigation is the method of conducting a plane along a course by radio aids, such as the radio beacon, radio direction finder or by radioed bearings. (The slang phrase "keeping on the beam," tells the story.)

IV. Celestial navigation is determining one's position by means of a bubble sextant from observations of the sun, moon, planets, and the stars, together with the exact time. Private or student pilots are not required to pass any academic test relative to celestial navigation for ratings as pilots. Therefore, this method is not taught in any elementary course.

Typical examinations and answers for pilot ratings are published by The Weems System of Navigation in a booklet entitled, "Civilian Pilot's Primer," for $1.00.

Any course should be carefully plotted, as to time, distance, and landmarks; then it is filed with the director of the port of departure before taking off. This implies, the reading and interpretation of charts which indicate topographical and aeronautical data relative to the relief of the land and the features of the airway in accordance with conventional symbols.

The distance of the proposed flight is found by measuring on a sectional chart and then applying the scale used in the construction of the chart. The time is estimated by assuming a speed of the plane for this distance. In general the time should be indicated for each ten mile intervals and also for each outstanding landmark. In terms of mathematics, the application of scale drawings solves the exercises for piloting.

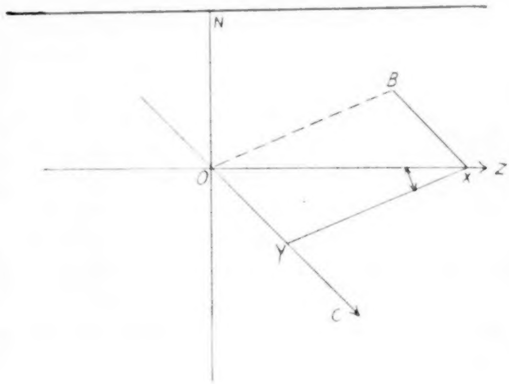
The direction of the flight is determined by the use of a compass course protractor. (However, important *corrections* are necessary before establishing the compass heading.)

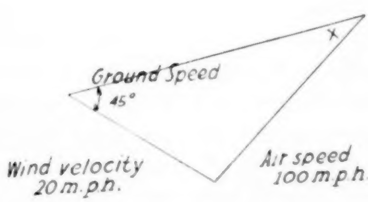
When planning a flight by dead reckoning, the pilot must determine from the chart the distance and the compass heading. The distance is again the application of scale drawing. The determination of the compass heading depends upon three variables: *Magnetic variation* (which varies with the port's location on the earth's surface), due to the fact that the magnetic pole is not identical with the true north pole, and may be read from a chart, *Magnetic deviation* (which varies with the plane due to local magnetism). *Effect of wind* known as the *Drift Angle*. Algebraic corrections for these variables although easy, lead to much confusion. The real mathematical problem is the drift of the plane due to the direction and velocity of the wind. This wind effect or drift angle may be calculated by three methods and then checked by a navigation computer. From the standpoint of physics this problem is an application of the parallelogram of forces or vectors. In terms of mathematics it is an application of triangulation; however, this problem may be solved by selecting a convenient scale and surprisingly accurate results may be secured by making a scale drawing of the triangle on graph paper.

I shall illustrate the graphic method by the following exercise:

Given: True course 90° East
Air speed 100 m.p.h.
Wind from 315° with a velocity of 20 miles per hour.

To find: the drift angle and ground speed.





(This solution should be on centimeter paper.)

1. The true course is represented by OZ 90° East.
2. With the protractor indicate the wind OC from 315°.
3. Assume the scale of 1 cm. = 10 mi.
4. Then construct OY equal to 2 cm.
5. Then 100 mi. would be represented by a distance of 10 cm.
6. With Y as a center and a radius of 10 cm. strike an arc cutting OZ at X.
7. Measure OX, which is about 11.3 cm. long, then in accordance with our assumed scale this represents 113 mi., the *ground speed*, of the plane.
8. Measure $\angle OXY$ which equals 8°, the *drift angle* caused by the wind.

($\angle XOB$ is the same angle.)

9. Neglecting other corrections, the compass heading would be 82°, $\angle NOB$ to make good a true course of 90°.

In air navigation this triangle is known as the triangle of *velocities*. To a teacher of mathematics, this problem is readily recognized as the solution of a triangle, having given two sides and an angle opposite one of the sides, to find the angle. Thus giving us a practical application of the *Law of Sines*. The ground speed may be calculated by the *Law of Cosines* or the *Law of Sines*.

The third method of solution of the above exercise depends on the ability of the student to read and interpret a series of graphs which appear on pages 147 and 153 of Bulletin #24 previously mentioned.

As a check, the computer illustrated on pages 142–144 of the same bulletin may be manipulated to secure an approximate answer.

Having illustrated how the drift angle is determined, may I return to the three variables considered when establishing the compass heading necessary to make good a true course. In aviation phraseology this is known as Case I, in *Dead Reckoning*.

When plotting a course we assume or compute the following data: The distance is 200 mi., the compass course protractor reads 271° which is technically known as the direction of the true course. Magnetic variation according to the charts for the port is 4° Westerly; the magnetic deviation of the plane to be flown is 1° Westerly; and the drift angle produced by a wind from the left is 7°.

The question for the pilot is: "How may I determine my *compass heading* to make good the proposed route if I consider the following three variables?"

1. Magnetic Variation 4° Westerly; 2. Magnetic Deviation 1° Westerly; and a Drift Angle of 7° due to a wind from the left.

The compass heading is determined by the proper use of the following formula:

Compass Heading = True Course + Correction for magnetic variation + correction for compass deviation + correction for drift angle.

Westerly variation and deviation must be added while easterly variation and deviation must be subtracted.

Drift angle from the right must be added but subtracted if from the left.

Applying the above data to the formula:

$$\text{Compass Heading} = 271° + 4° + 1° - 7°$$
$$= 269°$$

Thus far I have described only the planning or the plotting of the course before flight. After taking to his wings the pilot is confronted with the same problem but is in the *reverse order*. Then westerly variation and deviation must be subtracted while easterly variation and deviation must be added. Drift angle from the right must be subtracted but added if from the left. Thereby, the confusion, which leads to disaster. A recent case in point is the fact that the navigator of Rickenbacker's plane seemingly failed to correct for the proper drift angle. This opposite process is referred to as *Case II in Dead Reckoning*.

Before flight, he knew his true course and wished to establish his compass heading, but while in flight he reads his compass heading but must establish his true course to see if he is making good his course. He is also limited as to time. Why take 20 min. to establish your position above the earth's surface when traveling at the rate of 120 miles an hour and find yourself 40 mi. away from the position when the calculations are completed? Therefore, the need for computers and graphs to secure immediately approximate answers.

The flight of planes from an airplane carrier at sea implies a variation of dead reckoning which is referred to as *Radius of Action*. This problem includes the headings to be flown on the back. All of which is a solution of two problems similar to the one relative to dead reckoning and the

application of the following formula:

$$\text{Radius of Action} = \frac{(\text{ground speed out} \times \text{ground speed back})}{\text{ground speed out} + \text{ground speed back}}$$

This answer represents the distance per hour for the time available. The time being limited by the supply of gasoline. This problem has additional complications when the plane is to return to a moving base.

Radio navigation is chiefly dependent on following beacons and keeping on the beam or directing the plane in accordance with radioed bearings. If these fail, other methods of orientation must be used.

Celestial navigation is only used in long flights and as previously stated is not considered a part of an elementary course in navigation; however, it affords us an excellent application of spherical trigonometry.

If we are to properly prepare boys, I find we must teach a new trigonometric function, namely; the *haversine*, wich is defined as haversine $A = (1 - \text{Cosine } A)/2$. This function is used in naval navigation, which also necessitates the use of haversine tables.

Stated briefly, I have attempted to outline the equipment necessary and with an illustration demonstrate the academic nature of a course in Practical Air Navigation which I am hoping to see included as an elective course in our high schools and colleges not only to keep them flying, but to help keep mathematics in its rightful position.

Suggested Aids:

Publications of U. S. Office of Education

Pre-Flight Aeronautics in Secondary Schools: Leaflet #63

Pre-Aviation Cadet Training in High Schools: Leaflet #62

Technical Manuals by the War Department

Air Navigation—TM 1-205; 40¢

Mathematics for Pilot Trainees TM 1-900; 10¢

NAVIGATION ON AN ADVANCED LEVEL

Navigation of Air and Marine Navigation, by A. Day Bradly, American Book Co.

Educational Gains in War Are Stressed*

EDUCATION is more closely involved in this war than in any previous one in history and it is likely that education will be improved as a result, rather than wrecked, according to Dean Virginia C. Gildersleeve of Barnard College. Miss Gildersleeve was a speaker recently at the annual session of the National Association of Principals of Schools for Girls, held at the Hotel Biltmore in New York City.

She declared that "the fierce, glaring light of war" might burn away superfluities, leaving the essentials. She urged improvement in the teaching of mathematics and asserted that because the secondary schools had not taught mathematics and English very well "it is not impossible that because of this weakness our country may actually lose the war."

The war has uncovered the widespread unpreparedness of secondary students to go on in higher mathematics, and has also demonstrated the need for better English training and "the ability to understand facts and ideas conveyed through spoken or written words in our native language," she declared.

"Inefficiency and even disaster may result from the lack of it," she said. "Moreover, in the offices of government departments in Washington and elsewhere, and in essential civilian activities, confusion and waste and calamity are caused by vague, misleading thought and expression."

* From the New York *Times*, February 27, 1943.

Pre-Induction Courses in Mathematics*

Foreword

The United States Office of Education has received urgent and repeated requests from individuals and organizations throughout the country to give the secondary schools detailed suggestions for the teaching of mathematics for pre-induction purposes. In December 1942, the Office in cooperation with the President of The National Council of Teachers of Mathematics appointed a committee to make a survey of the mathematical needs of the armed forces and upon this basis to make a report concerning what the schools can do for the emergency. The committee consisted of Virgil S. Mallory, Professor of Mathematics, New Jersey State Teachers College at Montclair; William D. Reeve, Professor of Mathematics, Teachers College, Columbia University; Giles M. Ruch, Chief, Research and Statistical Service, U. S. Office of Education; Raleigh Schorling, Professor of Education, University of Michigan; and Rolland R. Smith, Specialist in Mathematics for the Public Schools of Springfield, Massachusetts, and President of the National Council of Teachers of Mathematics. Dr. Smith served as chairman of the Committee.

Post-induction training in the armed forces is based in considerable measure on manuals of instruction prepared especially for the technical occupations of warfare. It was the thought of the Committee that detailed analyses of a large number of these technical and field manuals would provide a first approximation to the mathematical needs of the military establishments, or at least afford a basis for evaluating the general character of the curricular modifications in secondary school mathematics dictated by the emergency. To this end, the Committee requested and received the fullest cooperation of the Army, the Navy, and the Civil Aeronautics Administration.

The Civilian Pre-Induction Training Branch, Industrial Personnel Division, Services of Supply, War Department selected approximately 50 of the Army instructional manuals covering occupations for which there is the greatest training need. These manuals were studied in detail and notes made of the various types of mathematics used, and of the mathematical background involved. In like manner the Committee analyzed about 2[illegible] Navy training manuals made available by the Training Division, Bureau of Naval Personnel, Navy Department. Similar consideration was given to the training materials of the Civil Aeronautics Administration. The mathematical needs of typical war production industries were also considered by examination of about 50 unit courses used in the Federal-State program of Vocational Training for War Production Workers conducted cooperatively by State Boards for Vocational Education and the U. S. Office of Education.

This report has been submitted to and approved by:

The National Policy Committee for the High-School Victory Corps

The Civilian Pre-Induction Training Branch, Industrial Personnel Division, Services of Supply, War Department

The Training Division, Bureau of Naval Personnel, Navy Department

The Civil Aeronautics Administration, Department of Commerce

John W. Studebaker
U. S. Commissioner of Education
Washington, D. C., February 1943

* Reprints of this report may be had at 10¢ each postpaid from The Mathematics Teacher, 525 W. 120th St., New York, N. Y. Larger quantities may be had at a reduction.

Reasons for this Report

It is expected that the United States Army will increase to at least 7,500,000 by the end of 1943. The Navy and Coast Guard will in this period be enlarged by several hundred thousand. The total of all the armed forces may ultimately exceed 10,000,000. This further expansion in a mechanized war will increase the present shortage of men and women trained in mathematics and in practical physical science, in both the armed forces and the supporting war industries.

Of the men inducted into the Army, a large majority must be given some technical training in either pre-induction or post-induction courses. The Navy requires an equally high per cent of skilled personnel. The Navy operates nearly a hundred post-induction schools, and the Army a far greater number. An analysis of the various technical manuals used in these schools provides convincing evidence that much time is now wasted teaching simple mathematical principles and skills to many men who ought to know these things when they are inducted.

Many of the jobs classified as technical in the Army require only simple arithmetic. However, if one takes the technical manuals in the post-induction courses as a criterion, some of the least technical jobs require a level of mastery of mathematics higher than that needed by such workers in civilian life.

The technical manuals of the armed forces show that there are many mathematical applications that only the military organizations should teach, but it should not be necessary for them to give instruction in elementary mathematics.

The problem confronting mathematics teachers of the high school is "How can instruction in mathematics be modified, and that immediately, so as to give the utmost aid in the emergency?" The crux of the problem lies in what can be done for boys and girls now in the last one or two years of the secondary school. For this reason, detailed suggestions have been given for pupils of this group who are not now studying mathematics. The time factors for the students receive due recognition in the recommendations which follow. General suggestions are given for those enrolled in the sequential courses.

Recommendations

The emergency need for boys and girls trained in mathematics has focused attention on the highly technical features of our mechanized civilization. The armed services and the supporting war industries need boys and girls trained in the proficient use of mathematics ranging from a real mastery of arithmetic fundamentals and such practical uses as are found in courses in general mathematics to the uses of higher mathematics in meteorology, ballistics, and other branches of science. Girls trained in mathematics are needed to replace men in industrial and other civilian positions which require the same range of uses of mathematics.

This range of uses of mathematics is very wide. Every high school pupil must be able to compute with assurance and skill and many will be called on to use the simple algebra, informal geometry, scale drawing, and numerical trigonometry of the right triangle now taught in most courses in general mathematics. A smaller number will be needed who have mastered all of the sequential mathematics of the senior high school and college. The range of uses of mathematics is as wide as the abilities of high school boys and girls. Intelligent guidance should guarantee that every high school pupil study mathematics according to his need and his ability. The four-year sequence in mathematics, including trigonometry and solid geometry, should be taken by those students who have a real interest in mathematics, who are capable of mastery of the subject, or who are likely to use mathematics in their further training and ultimate occupation. Counselors must realize the need for cer-

tain mathematical skills and understandings.[1] On the other hand, it is wasteful to put pupils into such courses if their aptitudes and abilities indicate that they cannot obtain a secure mastery of the material they are studying. It should be remembered in every case that unless the mathematics taught is mastered thoroughly, it will neither be of practical use nor will it serve the other purposes of mathematical study.

The demands of the armed services and war industries for pupils well trained in mathematics are of immediate concern. These demands are equally urgent in the armed forces and in industrial production. They need not, however, conflict with the needs of boys and girls for adequate training in mathematics for future civilian life. The emergency calls for more effective teaching of mathematics with adequate practice and practical applications to the end that pupils can use mathematics in a wide variety of practical situations. Mathematics learned as mechanical manipulation only cannot have its fullest value either in the immediate emergency or in future civilian life.

The type of subject matter, the time element in teaching, and the number of practice exercises must be so selected and modified that a maximum of understanding is secured. Accuracy and skill in application must be a main consideration, not the amount of material covered. Instead of special courses in mathematics, practical problems can be used both to motivate and to point out emergency uses. Recommendations are made later concerning material which can safely be eliminated from the sequential courses to assure this more effective teaching as well as to permit the introduction of immediate applications to fit the emergency. Good teaching of mathematics, modified as suggested, is more valuable than specialized mathematics courses given by teachers not qualified to teach them, or taken by pupils who do not have the necessary foundation in mathematics and science or the maturity for their successful mastery.

Some modifications in content of the sequential courses are advisable both for the war emergency and for future civilian uses. The mathematics taught should be practical to the extent that it has immediate application, that it is needed for other essential mathematics, sciences, or other advanced courses, or that it pertains directly to the war effort. The immediate needs for war service should be met by reduction in the amount of less important material and the substitution of material which is more essential.

Recommendations for courses in mathematics are not confined to boys. Girls are needed to replace men in all branches of industry and civilian life in which mathematics from its simplest elements to its more advanced study is used. The need is great for women to fill vacancies as laboratory technicians and assistants, as junior engineers, as workers in industry, and in the teaching profession. There is also immediate need in civilian work, in the signal corps and in industry for girls who are high school graduates.

Finally, the high school is confronted with the need for effective guidance of boys and girls so each will study that mathematics which he can master and use effectively. Three general categories of pupils are recognized for purposes of the specific recommendations which follow.

1. *Pupils with little or no mathematical background.* This group consists of boys and girls who have had not more than one year of training in mathematics beyond the eighth grade. If these pupils are near

[1] The following references will be useful to counselors and other guidance officers:

Victory Corps Series, *Guidance Manual for the High School Victory Corps.* In press. One free copy will be distributed to each high school when printed. Additional copies must be ordered from the Superintendent of Documents, Washington, D. C.

Minimum Essentials of the Individual Inventory in Guidance. By Giles M. Ruch and David Segel. U. S. Office of Education, Vocational Division, Bulletin No. 202. Occupational Information and Guidance Service Series No. 2. For sale by the Superintendent of Documents, Washington, D. C. Price, 15 cents.

graduation, induction, or employment, they should take work of the type suggested by the special one-semester course described on page 121. If they still have a year in school, they should take the special one-year course described on pages 117 to 121.

2. *Pupils who have studied mathematics for two years beyond the eighth grade but who are not now studying mathematics.* These pupils will ordinarily have had one year of algebra and one year of geometry. If they are near graduation, induction, or employment, they should take work of the type suggested by the special one-semester course described on page 121. If they still have a year in school, they should take the special one-year course (pages 117 to 121). For these pupils the one-year course can be more intensive than for those who have little background in mathematics.

3. *Pupils who are now enrolled in the four-year sequential mathematics courses.* The more capable of these pupils should continue in that work and should not substitute emergency courses in mathematics. The curricular modifications needed in the basic sequence in mathematics are those which will secure more effective teaching, permit the introduction of practical applications related to the war effort, and correct deficiencies in computational skill. Suggestions about material which may safely be omitted to make it possible to offer enrichment material and to introduce fundamentals of arithmetic and practical applications are given later. It should be realized that, under the impetus of the war, enrollments in this sequential work will tend to increase. This necessitates careful guidance and selection. These courses should be reserved for those who can pursue them with satisfaction and high achievement.

A Suggested Special One-Year Course

Because of the limited time available, the Committee had to choose between the alternatives of concentrating on the most critical phases of pre-induction courses in mathematics or attempting a full coverage of all phases of such training, thereby delaying publication so long that the report would have no usefulness during the present school year. The decision was reached to prepare a rather detailed outline of this special one-year course, leaving the way open to supplementary reports should the need develop.

The number of topics studied by any given class will depend upon the ability of the pupils. Many classes will be able to cover the entire outline. Mastery of selected topics is of greater value than superficial knowledge of this entire course.

Arithmetic

The importance of the fundamental arithmetical skills with integers, and fractions, both common and decimal, has been so generally accepted as a part of the training of every American citizen, that we should expect all pupils who go through our elementary schools to exhibit proficiency in such skills and their application. Many pupils, however, have been so poorly trained that there is a large number with little mastery of arithmetic. Such a situation should not be permitted to continue.

The fundamental processes with integers and fractions, both common and decimal, which are ordinarily taught in the first six grades of the elementary school furnish a basis for the real applications which should follow in the elementary and the secondary school. Arithmetic correlates well with algebra, informal geometry, and numerical trigonometry. All of these are important in solving wartime and peacetime problems.

A study of the various technical manuals suggests that both abstract drill and practical applications should be very simple. It is far better to build confidence and to strive for accuracy with simple illustrations than to confuse the pupils with involved computations that seldom,

if ever, occur in the actual jobs of the fighting forces. This suggestion applies to the work in algebra and geometry as well as to arithmetic.

A detailed outline follows:

I. Whole numbers
   1. Reading and writing large numbers
   2. Computation
      a. Addition of not more than six addends with not more than six digits in each
      b. Subtraction with not more than six-digit numbers
      c. Multiplication with the multiplicand containing not more than five digits and the multiplier not more than three digits
      d. Division with not more than three-digit divisors

II. Common fractions and mixed numbers
   1. Meaning (including the meaning of a fraction as an indicated quotient)
   2. Fundamental operations—Denominators should be mainly powers of 2 up to 64

III. Decimals
   1. Meaning
   2. Place value in numbers
   3. Fundamental operations (limitations similar to those under computations with whole numbers under I above)
   4. Changing common fractions to their decimal equivalents and vice versa. Ability to use tables of equivalents

IV. Per cents
   The work on per cents can be improved by practical illustrations from aviation and shop practice. In the post-induction manuals uses are made of all three cases of percentage. Practice should be principally on the first two cases: Finding a certain per cent of a number and finding what per cent one number is of another. The groundwork for teaching the second case should be laid by comparing two numbers as common fractions and as decimals. The topic of ratio and proportion affords an opportunity for refreshing the comparison concept

V. Averages (arithmetic mean and median)
   1. Meaning
   2. Computation (ungrouped data)

VI. Square root (see page 121)
   1. By division (trial and error)
   2. By table (interpolation)
   3. By the algebraic method

VII. Information graphs

VIII. Scale drawings (see page 120.)

IX. Keeping simple accounts

X. Applied problems

Informal Geometry

Geometry has a double value, first as knowledge and second, as a way of thinking. The importance of geometry in giving information of value in the daily life of every well educated citizen has been neglected, if not entirely overlooked. In making an object of any kind, one must first conceive the object in order to avoid waste of time and material. He must know the shape and the relation of the various parts, must be able to measure them, and finally, to fit (locate) them in their proper places in the completed object.

In building a special one-year course it must be decided what values of geometry are paramount for the national emergency, what content material should be selected and what methods should be used to facilitate its teaching. For this purpose, informal geometry should include:

> *Intuitive geometry*, where one looks at a figure and says that something is so because it could not be otherwise. Example, "If two straight lines intersect, the vertical angles are equal."
>
> *Experimental geometry*, such as cutting out a paper triangle, tearing off the angles and then placing them adjacent to show that their sum is 180°.
>
> *Observational geometry*, which consists of recognizing objects as typifying certain geometric forms, seeing certain relationships and important facts that exist between them, such as, "Any angle inscribed in a semicircle is a right angle," or, "If two parallel lines are cut by a transversal, the alternate interior angles are equal."
>
> *Geometric constructions* of a fundamental type, such as bisecting a line or an angle.

In this special one-year course teachers should realize that there will be little or no place for deductive proof. The early geometric knowledge of the race began with simple observations, intuitions, measurements of real objects, and recognitions of their relations to each other. The classification of such knowledge as the race had attained and the organization of the facts of their experience into a deductive science were outgrowths of man's earlier struggles. As with the race, so with the individual; the real experiences with the facts should come first and the more formal propositions and their logical proofs later. It is conviction in the minds of the pupils that should be sought. Few pupils can understand what they have not physically perceived.

In all of this work, the choice of content and the emphasis in instruction should be placed upon those topics which have a practical bearing in the ordinary affairs of daily life.

The content of this part of the course should consist of:

I. Geometry of form
  1. Shapes seen in nature
  2. Geometric concepts
    a. Point
    b. Line
      (1) Straight
      (2) Broken
      (3) Curve
      (4) Horizontal
      (5) Vertical
      (6) Parallel
      (7) Perpendicular
    c. Plane
      (1) Plane figures
        (a) Rectilinear — triangle (scalene, isosceles, and equilateral), quadrilateral (trapezoid, parallelogram, rectangle, square, and rhombus), and regular polygons
        (b) Curvilinear—circle, parabola, and ellipse
    d. Solid
      (1) Kinds
        (a) With plane faces—rectangular solid, prism, and pyramid
        (b) With curved faces—cylinder, cone, and sphere
    e. Angle
      (1) Measurement
      (2) Kinds
        (a) Acute
        (b) Right
        (c) Obtuse
        (d) Straight
  3. Similarity
    a. Types of similar figures
      (1) Triangles and other plane figures
      (2) Reduction or enlargement of similar figures
      (3) Photographs and maps
      (4) Scale drawings
  4. Congruent triangles (no proofs)
  5. Mechanical drawing
    a. Instruments to be used if they are available
      (1) Ruler graduated to thirty-secondths, to tenths of an inch, and to millimeters
      (2) Protractor
      (3) Compasses
      (4) Draftsman's instruments
        (a) Drawing board
        (b) T-square
        (c) Triangles (30°-60° and 45°-45°)
    b. Skills to be learned
      (1) Drawing angles with a protractor
      (2) Drawing parallel and perpendicular lines
      (3) Drawing rectilinear figures
  6. Constructions with ruler and compasses
    a. Circle
    b. Bisecting a line segment
    c. Bisecting an angle
    d. Copying an angle
    e. Dividing a line segment into more than two equal parts
    f. Constructing triangles, given
      (1) Three sides
      (2) Two sides and the included angle
      (3) Two angles and the included side
      (4) Two sides and an angle opposite one of them

II. Geometry of size
  1. Direct measurement
    a. Appreciation of the limits of accuracy in measurement and of the fact that all *measurement is approximate*
      (1) Limits of accuracy
      (2) Significant digits

(3) Rounding off to reasonable results
(4) Common sense checks

b. Estimating and measuring
(1) Lengths—using ruler, compasses, calipers, and squared paper
(2) Angles—using the protractor
(3) Area of regular and irregular figures using ruler, squared paper, and simple formulas
(4) Volume and capacity
(5) Weight—avoirdupois, Troy, and apothecary's
(6) Time, including the international 24-hour day

c. Applied problems

2. Indirect measurement
a. Scale drawing (commonly used in air and marine navigation)
(1) Reading and using scales
(a) Two forms: 1 in. =6 ft. or representative fraction (R.F.) 1/72
(b) Changing a scaled distance to its value in relation to the original object
(c) Determining proper scaled distance to represent an object
(d) Using grids (squared paper) to make scale drawings
(e) Using metric scales
(f) Using ordinary ruler marked with a scale
(2) Drawing geometric figures to scale
(a) Rectangles
(b) Triangles, given three sides, two angles and a side, two sides and included angle, two sides and the angle opposite one of the sides
(c) Using triangles (including oblique triangles) to solve surveying and navigation problems (See also under "use of vectors" below)
(d) Using lines drawn on a grid to solve distance, rate, and time problems
(3) Method of representing directions
(a) Mariners' compass
(b) Naval bearings
(c) Surveyors' bearings
(4) Use of vectors
(a) Using a line segment to represent by its length and direction a velocity or the amount and direction of a force
(b) Using vectors to solve problems
Finding the resultant of two forces
Finding heading and correction angle, given course, air speed, and wind velocity
Finding heading, given course, speed, and current
Finding wind velocity, given heading, ground speed, and drift angle
(5) Representation of front, top, and side views of simple objects.

b. Numerical trigonometry of the right trangle
(1) Development by a scale drawing of the tangent function for angles of 10°, 20°, 30°, . . . 70°
(2) Use of 3- or 4-place tables of tangents to solve right triangles given two legs, or a leg and an acute angle
(3) Development of sine and cosine functions by scale drawing for angles of 10°, 20°, 30°, . . . 80°
(4) Use of 3- or 4-place tables of sines and cosines to solve triangles, given hypotenuse and side, or hypotenuse and acute angle

Algebra

The main use of algebra is made in connection with literal notation, particularly in formulas and equations. Many topics generally taught in the sequential courses need not be taught in this special course. In particular the following topics should be omitted. Addition, subtraction, multiplication, and division of polynomials; special products; factoring, except the case of removing a common monomial factor; alge-

braic solution of simultaneous equations; quadratic equations, except the form $ax^2=k$; and so-called verbal problems. The level of difficulty of no topic should be greater than that required in commonly used formulas. This means that all topics should be held to the simplest examples.

A brief outline follows:

1. Symbolism
   a. Letters as symbols for numbers
   b. Symbols of operation including exponents, radical signs, and parentheses
2. Formulas
   a. Meaning
   b. Evaluating formulas concerned with shop, industry, and the armed forces including aviation (This requires only a knowledge of the meaning of symbolism and skill in arithmetic)
3. Equations
   a. Simple types such as $3n=6$, $n+3=6$, $n-3=6$, $\frac{n}{3}=6$, $2n+3=7$, and $2n-3=7$
   b. Indirect use of the formula (substitution of given values and solution of resulting equation for the unknown)
4. Signed numbers
   a. Meaning and use
   b. Fundamental operations
   c. Equations which can be reduced to the form $ax+b=cx+d$ including those having a single parenthesis
5. Graphs
   a. Familiarity with the coordinate system
   b. Interpretation of many kinds of simple graphs
   c. Graphing linear formulas
   d. Solution of simultaneous equations (The intersection of two lines frequently furnishes the solution of problems in navigation)
6. Operations (only the simplest examples)
   a. Combining like terms
   b. Laws of positive integral exponents
   c. Removing parentheses
   d. Multiplying a polynomial by a monomial
   e. Division by a monomial
   f. Factoring (One case, taking out a common monomial factor)
   g. Algebraic fractions (No more difficult operations with fractions or fractional equations need be taught than those that are necessary for evaluating commonly used formulas)
   h. Ratio and proportion
   i. Radicals
      (1) Square root by computation
      (2) Square root by table (interpolation)
      (3) Simplifying radicals only as needed in using tables
      (4) Use in formulas
   j. Quadratic equations of the form $ax^2=k$

In the use of notation recognition should be given to the fact that many practical formulas make use of both capital and small letters and letters with subscripts, primes, and superscripts. In substitutions both common and decimal fractions as well as integers should be used. In equations both common and decimal fractions should occur as coefficients.

Special One-Semester Course

This course is an emergency refresher course for high school pupils who are near graduation or induction but who are not at present studying mathematics. It should be realized that mathematical knowledge which the pupils do not possess cannot be refreshed. Hence, the material used in this course will depend on the previous training or lack of training of the pupils. For everyone it will contain the fundamentals of arithmetic after diagnostic tests have determined the needs of the pupils. For some, those with little or no mathematical background, this arithmetic plus certain essential topics from general mathematics such as scale drawing, including elements of blueprint reading, numerical trigonometry, informal geometry, and simple formulas and equations, will constitute the course.

For pupils who have had previous courses in algebra and geometry the refresher course should emphasize those aspects of mathematics that are particu-

larly applicable and essential to the war effort. It should begin with the fundamentals of arithmetic. The review of algebra and geometry should reduce emphasis on operations with polynomials (eliminating long division completely)

The following should also be eliminated:

Special products and factoring (except monomial factors, and possibly the difference of two squares and the perfect trinomial square)

Fractions with other than monomial denominators

Complex fractions (the type of fraction retained should be that found in formulas in geometry, physics, and simple shop situations)

Equations containing artificial fractions

Complex work in radicals (the types of radicals retained should be those occurring in geometry, numerical trigonometry, and physics)

Quadratic equations (except those of the type $ax^2=k$)

Deductive logic in the establishment of geometric relations

In the small high school it may be necessary to teach pupils of varying backgrounds in the same classes. In the larger high school at least two situations should be recognized:

1. A course for those pupils who have had no high school mathematics or only a one-year course in general mathematics or algebra. This should contain, in addition to the first topic below, as many of the other topics as can be done effectively.

(a) Fundamentals of arithmetic to correct deficiencies discovered by diagnostic tests

(b) Topics from general mathematics including scale drawing leading to the elements of blueprint reading; numerical trigonometry and uses of the Pythagorean Theorem; informal geometry including constructions, congruence, and similarity; use of tables

(c) Many practical applications from aeronautics, navigation, and shop uses of mathematics[2]

2. A course for those who have had two years of mathematics. This should contain as many of the following topics as can be done well:

(a) Fundamentals of arithmetic to correct deficiencies discovered by diagnostic tests

(b) Review of previous courses in mathematics omitting deductive derivation of geometric theorems and many of the purely manipulative and highly complicated expressions in algebra as indicated previously

(c) Many applications from aeronautics, navigation, artillery fire, parallelogram of forces and triangle of velocities, shop uses of mathematics; use of tables and the slide rule

Sequential Courses

The sequential work includes four years of mathematics beyond the eighth grade (seventh grade in states with a seven-grade elementary school) and should include solid geometry and trigonometry. Schools which find it possible may offer for the more interested and capable of these pupils additional work in mathematical analysis including topics selected from advanced algebra, spherical trigonometry, and the elements of analytical geometry and the calculus.

Each year of the course should contain some work in the fundamentals of arithmetic to insure increasing speed and accuracy in computation. Even competent pupils enrolled in the sequential courses do not always have adequate skill in applying fundamentals to whole numbers, fractions, decimals, and per cents. In many schools the courses in algebra and geometry fail to maintain arithmetic skills. This work

[2] A fertile source of both wartime and industrial applications is the Seventeenth Yearbook of The National Council of Teachers of Mathematics, *A Source Book of Mathematical Applications*. Bureau of Publications, Teachers College, Columbia University, New York, N. Y., 1943. Price $2.

should not be aimless but should be skillfully selected practice to correct deficiencies made evident through diagnostic tests. This work in arithmetic has already been outlined in connection with the special one-year course.

Many practical applications from physics, shop and industry, engineering (including use of the slide rule), navigation, artillery fire, and the parallelogram of forces including the triangle of velocities should be introduced. In selecting the applications there is need to guard against choosing those in which the technicalities are too great. A selected bibliography of such material is given at the end of this report.

Modifications in content to permit of more careful teaching and of the introduction of practical applications are possible. If an attempt is made to cover too much material, pupils will not receive sufficient practice to secure reasonable mastery of the subject.

Suggestions for modification follow:

Algebra. Reduce the amount of time spent on special products and factoring, on complex fractions, on fractions with other than monomial denominators, on equations containing such fractions, and on complex work in radicals.

Increase the amount of time spent on fundamentals of arithmetic, numerical trigonometry, on the use of practical formulas in industry, aeronautics, and science, and in the solution of practical problems in those branches.

Plane Geometry. The objectives of a course in plane geometry should be to acquaint the pupil with geometric facts and their application, and to give him an appreciation of postulational thinking. It is not necessary to demonstrate all of the propositions in a deductive course. Some can be treated informally. By wise selection of theorems to be proved, time will be provided for the fundamentals of arithmetic and for continued use of algebra as well as for such practical problems as: "doubling the angle on the bow" and "bow and beam sailing," radius of action problems, the parallelogram of forces and the triangle of velocities, numerical trigonometry, the extension of locus to include the derivation of the equations for the circle, parabola, ellipse, and hyperbola, the law of sines and cosines, and the use of the transit and other instruments.

Plane Trigonometry. Omissions may be made in the development of trigonometric analysis including the proof of general trigonometric identities and the solution of trigonometric equations. Pupils definitely pointed toward higher mathematics will, however, need some work with equations and identities.

Solid Geometry. The proofs of formulas for surfaces and volumes can be treated informally. Deduction should be limited to the relations between lines and planes in space and to figures drawn on the sphere. Theorems about the intersection of two planes and all of those theorems about perpendicular lines and planes and parallel lines and planes are particularly important for an understanding of three-dimensional space. Relations of figures on a sphere lead directly to terrestrial geometry and should be emphasized. Small circles and great circles should be connected with circles of latitude and meridians of longitude on the earth's surface and with great-circle sailing.[3]

Time thus saved in both trigonometry and solid geometry can profitably be spent on improving skills in computation, on practical applications including the use of the slide rule, elements of navigation, or on mathematical analysis including topics

[3] It is interesting to note that this recommendation concerning solid geometry is supported by a recent statement of the College Entrance Examination Board with respect to its new Comprehensive Mathematics Test. "This examination will test the candidates' knowledge of algebra through progressions, plane geometry, trigonometry and logarithms, and intuitive [informal] and computational solid geometry. It is designed for students who wish to enter courses in engineering or other sciences, which have mathematics as a prerequisite. Candidates for this examination will normally have had four years of mathematics beyond the eighth grade."

from analytic geometry, the calculus, or advanced algebra. These topics, however, should not be introduced to the detriment of a clear understanding of elementary work.

Suggestions to Teachers

Teachers of mathematics whose academic and professional training has been adequate will need further training to carry out the recommendations in this report as follows:

1. Study of wartime and industrial applications of the fundamental ideas of mathematics
2. Further training in science in order to correlate mathematics properly with science
3. Refresher courses in colleges and universities when necessary

Teachers in other fields who will be inducted into the teaching of mathematics because of the emergency, and there may be a considerable number of them, should in every way seek to prepare themselves to do as acceptable work as possible. Those who have little or no knowledge of what they are to teach cannot expect to be successful. The main methods for improvement consist of:

1. Attendance at summer schools and in part-time courses especially planned to meet their needs. Such courses should include professionalized subject matter in the content material which they have to teach
2. Departmental meetings where problems of the emergency should be discussed (in the larger city school systems experienced teachers of the departments of mathematics should feel obligated to give as much help as possible to the less experienced ones)
3. Studying the content of elementary mathematics
4. Studying the best literature on the teaching of mathematics. A short selected bibliography is given which should be helpful to those who need it
5. The teacher of physics and chemistry will be able to suggest the types of formulas and equations used in applied science
6. In the larger school systems having vocational schools or departments, the instructors in trade and industrial education can be of a great deal of assistance to the inexperienced teacher by suggesting industrial applications of mathematics

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The following bibliography is not intended to be exhaustive, but suggestive of practical applications that may be helpful to teachers of mathematics and science. It includes only the more elementary works. Some examples and some of the developmental material in many of these books are open to criticism, but thoughtful selection will yield a large amount of helpful material. For obvious reasons no attempt was made to list all of the useful textbooks.

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Practical Air Navigation. Civil Aeronautics Bulletin No. 24. Superintendent of Documents, Washington, D. C.

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Some Military Applications of Elementary Mathematics. Prepared by the Department of Mathematics, U. S. Military Academy, West Point, N. Y. The Institute of Military Studies, University of Chicago, 1942. Price, 15¢.

Suggested Unit Course in Blue Print Reading for Beginners in Machine Shop Practice. Also a Teacher's Manual and Answer Book for the same. The State Education Department of New York, Albany, N. Y. Price, 80¢.

Suggested Unit Course in Mathematics for Beginners in Machine Shop Practice. The State Education Department of New York, Albany, N. Y. Price, $1.35.

Poems*

By June Myers
Student, University of Chicago High School

THEOREMS

Theorems are what we come to now,
They must be proved and it matters how.

Whatever happens to mice and men,
A theorem must contain an "If" and a "Then."

The proofs always look so nice and easy(ble)
That it's quite natural it should seem feasible,
That we should have the pleasant illusion,
That we're only a step from our conclusion.

But to a weakened structure they act as a serum.
So, hail to thee, O Mighty Theorem!

UNDEFINED TERMS

If you want your forehead to be clear and unlined
Don't ever wrestle with terms undefined.

If you're slightly dumb, or mentally lazy,
Don't torture yourself with conceptions hazy.

I've a solution that worked out for me;
It's very simple and easy to see.
I make my list as short as can be,
And say I see them intuitively!

CONSTRUCTIONS

Whenever my eraser is worn out and weary,
Whenever my eyes are tired and bleary,
Whenever I'm frantically trying induction,
You can bet I'm working on a construction.
My arcs don't arc, my segments are merely a guess,
I'm completely in the dark for a way out of this mess,
And if my spheres are even slightly spherical,
All I can say is, "It's a miracle!"

POSTULATES

These are things we accept to be true.
And we believe them most thoroughly, too.

Gems of wisdom that have been doled me,
These are the things the author done told me.

Although postulates are handy, we scarcely love them
And like them better the less we have of them.

SYMBOLS

There are tricks in every trade
That never fail to come to our aid.

The seamstress has her shining thimble.
The mathematician his little symbol.

DEFINITIONS

Some say our definition list is far too long,
But its place in our structure is very strong.

And if ever you, your definitions change
You get some statements very strange.

We have had a more than goodly number
To haunt us and disturb our slumber,
For there's one thing you can't see by intuition,
You've got to learn a definition.

* These poems were sent to The Mathematics Teacher by Miss Martha D. Alexander, who secured Miss Myers' permission to publish them. They were used by Miss Myers as introductions to various sections of her geometry notebook.—Editor.

◆ THE ART OF TEACHING ◆

Teaching Solid Geometry

By Nancy C. Wylie
Dobyns-Bennett High School, Kingsport, Tennessee

This article is corroborating and supplementing the timely suggestions made by James V. Bernardo in the January 1940 issue of The Mathematics Teacher, on the teaching of solid geometry. I say, timely; first, because of its contribution to the meager body of material on the teaching of these books of Euclidean geometry; second, because most instructors are ready to begin a new semester and desire all additional light on methods of presenting the three-dimensional concepts. Any aids for perfecting technique that will enable the instructor to help the pupil in developing his spatial imagination will, very probably, be received with enthusiasm.

After presenting solid geometry for several years, with little hope of even the best student beginning to visualize the figures until well along in the fifth "book," I, too, conceived the idea of having models made of the figures. The student, with the aid of cardboard, twine, an old pencil, or, a lollypop stick (as mentioned in Mr. Bernardo's article) makes a model. Then, there is no doubt as to the position of a point or a line. The student sees at once whether it is on a plane or outside the plane. Prior to the use of this method of instruction, students seemed unable to lift their thoughts above the plane.

Let us consider the theorem: "A line perpendicular to each of two intersecting lines at their point of intersection is perpendicular to their plane." The explanation of this theorem by use of a cardboard figure results in immense satisfaction for both student and instructor. The same method of procedure is used by the writer as that suggested in "A Helpful Technique in Teaching Solid Geometry" by Mr. Bernardo referred to above. Blackboard figures, where, of course, three dimensions must be presented on a two-dimensional surface, accompany the cardboard figures; and the student enjoys the challenge, for it is "up to" him to explain the position of each point and line. He takes great pride in giving a perfect demonstration, and becomes quite proficient in handling the barrage of questions.

In this theorem, the model is most useful in explaining why lines of the same length, to give that appearance, are not always made the same length in a drawing on a plane surface. Often, a right angle is represented by drawing an acute angle. These simple facts, after a year of plane geometry (working with compasses, in two dimensions) are most confusing to a student beginning a study of a subject which makes use of three dimensions. Once the student understands the meaning of perspective, the voyage into three dimensions is not one into utter darkness.

"At long last" we, too, come to the five regular solids. To say there is satisfaction in completing the *Platonic Bodies*, all mounted and labeled, ready to be placed behind glass doors for the enjoyment of the entire mathematics department—well, that is like saying da Vinci was satisfied when his brush made the final touch on "The Last Supper," or that Beethoven

was happy when he had composed a symphony, or completed a sonata. Here students have constructed with mathematical precision the only five regular solids possible, known even to the Pythagoreans. Euclid has proved that there are only five! Plato studied and taught these regular solids! They bear his name! Here the student has himself made the tetrahedron, hexahedron, octahedron, dodecahedron, icosahedron, linking himself with old Greece, and making further contact with eternal truth.

We have reached a high plane, only to find that, as in life, we cannot remain on the mountain top. So this, too, drops back into the prosaic. We have not thus solved all our problems. In due time, this theorem appears: "The volume of a triangular pyramid equals one-third the product of its base and its attitude." By now we have no difficulty in visualizing, and have long since abandoned the practice of making a model. "But, Miss A, where is that triangular pyramid C—AGB?" Then a versatile student resorts to ivory soap, and in a humorous vein, informs this instructor that he thinks she will be surprised to see what that third triangular pyramid looks like.

Even so, we still have that student for whom the lines in the figure simply do not "come up." He cannot see three distinct solids. Then, we again resort to our *Diagrams in Three Dimensions.*[1] We had such fun looking through the spectacles that first day! At various times when the figures would not "come to life" we have used them. This time, with our diagrams and red and blue spectacles, they stand out so distinctly that we wonder how we ever failed to find them.

We cannot be completely satisfied. Yes, we want to see inside the figures, so we use our celluloid figures.[2] It is much better to look into that regular polyhedron circumscribed about a sphere, and—yes, there the sphere is—inside there, sure enough. Now, prove: "The volume of a sphere equals one-third the product of its radius and its area."

With no attempt at being facetious, where are those who say mathematics is cut and dried? One might think that they long since would have silently stolen away, without waiting to fold their tents. We are not deluded. An elementary study of probabilities will show that, on the wheel of time one will come around every —th time. The number should, however, be infinitesimally small, so that, like a part of the formula in the infinite series, it is negligible.

[1] By E. R. Breslich.

[2] Ruth W. Stokes, Professor of Mathematcis, Winthrop College, Rock Hill, South Carolina, is actively interested in building models for use in teaching mathematics.

Notice to Subscribers

Earlier in the year some material was sent out to some members of the National Council of Teachers of Mathematics by the Navy Department. Since then we have had inquiries from other members asking for this material. The Navy Department announces that some of this material is available, namely:

1. Teaching Aids Sample Problems and Sample Examinations Taken from Courses Being Taught at Naval Training Schools and Aviation Bases Covering Aeronautics Communications, Mechanics, Navigation and Other Subjects. No longer available.
2. Fifty Typical Problems and Examinations. Available from Mr. William Martin, U. S. Office of Education, Washington, D. C.
3. Refresher Course in Fundamental Mathematics. Available from the Naval Institute, Annapolis, Maryland.

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* Mr. Kenneth Brown former Chairman of State Representatives is now in the Navy. His successor has not yet been appointed. Vacancies in Arkansas, Idaho, New Mexico, and South Carolina will be filled as soon as possible.—Editor.

EDITORIALS

Professor Earl Raymond Hedrick

Professor Earl Raymond Hedrick, well known mathematician and teacher and member of the faculty of Brown University Graduate School in Advanced Instruction and Research in Mechanics, died in Jane Brown Hospital in Providence, R. I. on February 3, 1943 at the age of 66 years.

Professor Hedrick was originally professor of mathematics at the University of Missouri. He later went to the University of California at Los Angeles as Professor of Mathematics. More recently he has been Vice-president of the University there where on July 1, 1942 he was retired at the stipulated age of 65.

He was a member of The American Mathematical Society, The Mathematical Association of America and The National Council of Teachers of Mathematics. He was active in all of these organizations. Along with men like Professors H. E. Slaught and U. G. Mitchell, he was a great friend of the teachers of secondary mathematics.

A few weeks before Christmas last year he underwent a serious operation, but from all accounts he was recovering when the end came rather unexpectedly. Professor Hedrick's passing is a great loss not only to his family and his many friends but also to mathematical education in this country. Of course we could not expect to have him with us always, but his work certainly was not finished. He was on the threshold of doing many things for mathematics in which he was interested and which a man of his ability and temperament could have accomplished.

W. D. R.

Pre-Induction Courses in Mathematics

This issue contains the final report of a committee on "Pre-induction Courses in Mathematics." This committee was organized by the U. S. Office of Education in collaboration with Dr. Rolland R. Smith, president of The National Council of Teachers of Mathematics, who served as chairman. All of the members of the committee are also members of The National Council of Teachers of Mathematics. It is a great tribute to the Council that the U. S. Office of Education should ask its president to collaborate in such an important piece of work. If there is one organization in the country that should be ready to be called upon to render service in formulating a program in mathematics for secondary schools during the emergency, it is The National Council of Teachers of Mathematics.

It will be a noble gesture, now that the report is available, if members of the Council will give it all the publicity possible so as to assist the Office of Education in getting the report broadly circulated.

The Mathematics Teacher appreciates greatly the privilege and the honor of publishing the report for the U. S. Office of Education.

W. D. R.

IN OTHER PERIODICALS

By Nathan Lazar
Midwood High School, Brooklyn, New York

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4. Barrow, D. F., "Can a Robot Calculate the Table of Logarithms?" pp. 671–673.
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3. Finkel, Benjamin F., "A History of American Mathematical Journals," pp. 21–30.
4. Boldyreff, A. W. and Hohn, F. E., "On the General Definition of a Conic," pp. 31–37.

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2. Kenney, J. F., "Characteristic Functions in Statistics," pp. 51–67.
3. Farnell, Albert B., "Summation of Finite Series with Polynomial Terms," pp. 68–69.
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NEWS NOTES

New Jersey Meeting

Because of transportation difficulties and the Army occupation of Atlantic City, the seventy-sixth regular meeting of the Association of Mathematics Teachers of New Jersey was held at the Hotel New Yorker, New York City, on Saturday, November 21, 1942.

The large number of teachers attending the meeting seemed to indicate that the mathematics teachers of New Jersey are well aware of the problems facing them now and in the future. The choice of speakers and topics pointed up these problems.

Professor Albert E. Meder, Jr., of New Jersey College for Women, in his "Report of Arithmetic Knowledge of College Freshmen," concluded that such knowledge was woefully lacking. On a test which included common fractions, decimals, per cents and some verbal problems, 123 out of 300 entering college students made scores of less than 55 per cent. The test was given without preparation or review. The easiest problems seemed to be those involving common fractions; the hardest, those involving percentage or reasoning. Professor Meder is attempting to meet this problem by requiring a refresher course for all those who show the need for it.

The next speaker on the morning program, Dr. Edna E. Kramer of Thomas Jefferson High School, Brooklyn, showed the "Construction of Triangles in Air Navigation." By using the wind velocity, the air speed and course of a plane as sides of a triangle, other factors involved in flight may be found. These applications fit in neatly with congruent triangles in geometry and with scale drawing. They may also be used as problems in vector analysis and addition.

Dr. Henry Kentapp, Superintendent of Schools of East Orange, New Jersey, pointed out the "Causes of Deficiencies in the Mathematical Ability of High School Graduates." He declared that neither the time given to arithmetic nor conditions of teaching it are suitable to mastery of the subject. Also, mastery at the time of learning arithmetic does not presuppose remembrance. Therefore, what has been blamed on poor teaching might better be blamed on lack of memory and use. Certain corrections for this condition are now being made. First—many high schools are now giving refresher courses in arithmetic fundamentals and comprehension. Second—advanced courses are given to those who need it for the war effort. Third—war problems are being added to regular mathematics classes.

Dr. Carl N. Schuster of the State Teachers College at Trenton, N. J., upon invitation of Mrs. Florence Gorgens, president of the Association, made a few remarks. He emphasized that the Army does not wish schools to teach aeronautics at the expense of regular mathematics and science courses. Army authorities ask, rather, that mathematics and science be taught well and that aviation and navigation be left to the authorities on these subjects.

After a short discussion period, the members adjourned to the dining room for luncheon.

After luncheon, Mrs. Gorgens introduced Mr. Charles Philhower, president of the New Jersey Education Association, who extended greetings to the members.

The first speaker of the afternoon was Dr. Willy Feller of Brown University, Providence, R. I. whose topic was "On Applied Mathematics." Dr. Feller believes that much of the traditional mathematics should be redesigned for more direct use; that there are many applications which might better be used in place of the forced ones used so much today.

Lt. A. D. Kagon, of the United States Navy, gave the Navy viewpoint on "Mathematics or Frills." He quoted his Commanding Officer as saying that his whole speech might be summed up in three words, "Work Harder, Friends!" He asked that, if the schools have teachers qualified to do so, they teach only up to but not including celestial navigation. It is the opinion of the Navy that, before a student is ready for elementary navigation, he must know the following thoroughly:

1. four arithmetic fundamentals
2. algebraic solution of a right triangle
3. in trigonometry, the sine and cosine of right triangles
4. use of logarithms and interpolation
5. decimals
6. proportion
7. simple equations.

These talks were also followed by a short discussion period before the meeting was adjourned.

The committee in charge was composed of

General Chairman—	Hubert Risinger
Program—	Dr. D. R. Davis
	Dr. Amanda Loughren
	Dr. Virgil S. Mallory
	Miss Mary Rogers
Tickets—	Mrs. Dorothy Frapwell
Press—	Dr. Fred L. Bedford

Metropolitan New York Section Mathematical Association of America

A meeting of the executive committee of the Section was held on Saturday, December 19, at the Faculty Club of New York University, 22 Washington Square North, New York City. The purposes of the meeting were to decide on the time and place of the annual meeting of the section, to formulate plans for the program for the annual meeting, and to consider any other questions relevant to the purposes of the section.

The by-laws of the section provide that: "There shall be an executive committee composed of the officers, a representative of each collegiate department of mathematics in the metropolitan area which accepts an invitation to name a representative, a representative of each association of high school mathematics teachers in the metropolitan area, and a representative of science and industry to be selected by the officers."

Harris F. MacNeish, Brooklyn College, Chairman
Edna E. Kramer-Lasser, Thomas Jefferson High School, Vice-Chairman
Howard E. Wahlert, New York University, Secretary
Frederic H. Miller, Cooper Union, Treasurer.

The Mathematics Section of the Louisiana Teachers Association (affiliate group of the National Council) had a very fine meeting in Shreveport on Tuesday, November 24, 1:30 P.M. There were approximately seventy-five present. Listed below is the program of the meeting:

Officers

Miss Carolyn Rosenthal, Baton Rouge .... President
Miss Irma Smart, New Orleans .... Vice-President
Miss Melva LeBlanc, Kaplan .... Secretary

Program

Theme: "Coordination of teaching of elementary school, high school, and college mathematics."

1. Music.
2. Mrs. Olivia H. Shanks, Ouachita Parish High School, Monroe, representing secondary mathematics.
3. Professor P. K. Smith, Louisiana Polytechnic Institute, Ruston, representing college mathematics.
4. Mr. John B. Robson, State Supervisor of Mathematics and Science, Baton Rouge, representing State Department of Education.
5. Open discussion led by Dr. Houston T. Karnes, Louisiana State University, Baton Rouge.
6. Business.

The following officers were elected for the coming year:

President: Mrs. Frances O'Neal, 439½ Ruston Street, Bossier City, Louisiana.

Vice-President: Miss Lucille Brown, Fairpark High School, Shreveport, Louisiana.

Secretary: Miss Ruby Dry, Box 275, Logansport, Louisiana.

The Women's Mathematics Club of Chicago and Vicinity held their second meeting on Saturday, November 14, in the Charter Room of the Chicago Bar Association. Miss Hortense E. Wikart, a former mathematics teacher, and now of the Patent Research Department of Crane Co., gave a talk entitled, "Yankee Ingenuity." Cadet Wm. F. Lytle of the Army Air Force, located with the Training Detachment at the University of Chicago, also talked giving the Mathematics Requirements for the Army Air Forces.

Ionia J. Rehm, Publicity

The Association of Teachers of Mathematics in the Middle Atlantic States held its annual meeting at the Hotel New Yorker in New York City on Saturday, November 28, 1942.

Lyttleton B. P. Gould, Special Assistant to Chief of Naval Personnel, spoke on "Navy Requirements in Mathematics."

Samples of mathematical material used by the Navy were exhibited by Elizabeth H. Wood of the Girls' High School, Philadelphia, Pa.

There was a discussion of the report of Regional Committee on Science and Mathematics.

President—Herman P. Breininger, Germantown Friends School.

Secretary—Ruth Wyatt, Woodrow Wilson Junior High School, Philadelphia.

The Connecticut Valley Association of Mathematics Teachers met at Smith College in Northampton, Mass., on Saturday, November 7, 1942. About 75 were present and a very enthusiastic group they were.

Program

Morning

1. Mathematics and the War. Professor J. R. Kline, University of Pennsylvania.
2. Some Military Applications of Mathematics. Mr. Winfield Sides, Phillips Academy.

Afternoon

3. Long Needed Changes in the Mathematics Curriculum Hastened by the War Emergency. Mr. Goron Mirick, Lincoln School, Teachers College.
4. Pre-flight Aeronautics in Secondary Schools. Mr. Benjamin R. Brishey, Educational Consultant, C.A.A.

DOROTHY S. WHEELER

The third meeting of the Men's Mathematics Club of Chicago was held on Friday, December 18, 1942 at the Central Y.M.C.A. at 6:15 P.M.

Lieutenant Commander E. A. Beito, Navigation Department, Abbott Hall, Chicago, spoke on the topic "Mathematics and Navigation in the Navy."

Commander Beito saw considerable service at sea in the last war; before coming into the Navy again he was Professor in charge of Advanced Mathematics at the University of Wichita, Wichita, Kansas.

Mr. Homer C. Torreyson of Lane Technical High School spoke on the topic "Mathematics in a War Program."

The Mathematics Section of the Illinois High School Conference of the Illinois Association of the National Council of Teachers of Mathematics met at 2 P.M. on November 6, 1942, in 300 Mathematics Building, Urbana, Illinois.

The meeting was called to order by the Chairman, Mr. W. W. Willis, Dupo, Illinois. He introduced Dr. Ayres who urged the teachers to join the National Council of Teachers of Mathematics and benefit by its official organ, THE MATHEMATICS TEACHER. He also had the year books for sale. Mr. Nelson, Decatur, Illinois, announced the Science and Mathematics meeting at Chicago, Illinois, during Thanksgiving week-end.

Miss Ruth Siebert, Dupo, Illinois, gave the report of the nominating committee: Mr. Walter Barczewski, Waukegan, Illinois, for Chairman, Miss Lois Busby, Danville, Illinois, for Vice-Chairman, Miss Lucy Glascock, Eldorado, Illinois, for Secretary. There were no nominations from the floor. The Secretary was instructed to cast a unanimous ballot for the "slate" as read.

Mr. S. M. Runninger, East Aurora, Illinois, spoke on "What the High Schools in Northern Illinois Are Doing to Meet the War Demands." His remarks were based on a questionnaire which he had sent to 100 Northern Illinois high schools. He received 82 replies. The results of the questionnaire were:

3 schools—no effort made
2 schools—extra curricular only
25 schools—additions to old courses
52 schools—new courses
36 schools have a course in aeronautics.
16 use Elements of Aeronautics by Pope & Otis, World Book Company, as a text.
15 use Pre-flight Aeronautics published by Macmillan Company.
5 use other text material.
27 number of science teachers teaching aeronautics.
15 number of mathematics teachers teaching aeronautics.
1000 students in 82 high schools being trained in some sort of pre-flight aeronautics.

The country is calling on our schools to help in supplying pupils with a knowledge of aeronautics, but competent teachers are lacking in many schools. There has been a decided change in attitude toward mathematics and science—they are now recognized as important subjects.

In Aurora a class in geometry is being taught without the prerequisite of one year of algebra. No work in formal proofs is done. This class is being trained for industry and not for college.

Mr. W. O. Simmons, Chester, Illinois, told us "What the High Schools in Southern Illinois Are Doing to Meet the War Demands."

a) Classes in pre-flight Aeronautics.
    1) One-half of them being held after school hours.
    2) One-half of them without credit.
b) Military drill.
c) Other departments besides Mathematics and Science are making changes to further war demands.
d) Including Spanish and the history of Latin American countries in the curriculum.
e) Changing the wording of thought problems to fit present day situations, both in Algebra and Geometry.
f) More field work in Trigonometry.
g) More stress on drill in the fundamentals.
h) Physics classes adding units in aeronautics.
i) Refresher courses in Mathematics—teachers tutoring prospective servicemen after school hours.
j) Cooperating with various agencies in war demands—such as Red Cross, rationing, etc.

In a discussion from the floor the following points were made:

a) Most schools require Mathematics as a prerequisite for the Aeronautics course.
b) If the high schools give the basic elements in Mathematics, the Navy and Army will be able to go ahead with basic Officer training.

c) Girls should be permitted to take the Aeronautics courses.
d) Women, who have had Mathematics training through calculus, are in demand to such an extent that the University of Illinois cannot supply the demand.
e) Evening classes are being held in many high schools for adults who wish shop work, refresher courses, etc.

Captain Ned L. Reglein, Supervisor of the Training Films Preparation Unit, Chanute Field, Rantoul, Illinois, spoke on "High School Mathematics and Aeronautics." He pointed out that since air power is deciding the fate of nations, the schools should urge their students to take courses in aeronautics. Also, teachers of mathematics should use the formulas of aviation in their work on formulas, and aviation problems wherever they can be fitted into the work. By providing interesting applications, mathematics can be made a glamour subject. He suggested that teachers completely revise mathematics courses with aviation as the core or include aviation problems in the normal course. All education must become air-minded.

Mr. Raleigh Schorling, University of Michigan, talked on "Some Trends and Some Specific Changes in the Teaching of Mathematics Relating to War Demands." He said the teacher of mathematics is now much in demand and has a crucial job. More students will be enrolling in mathematics courses, since this is a war of physics and mathematics. We must gear our work to the war effort, and can do this better if we are given specific patterns with which to work. The student who has a major in mathematics is already prepared. There would be more math majors, if in the past ten years there had been less pressure brought to bear on the teaching of mathematics in high schools and colleges.

Mr. Schorling stated that we need a job analysis of induction units. Our directions must be more definite. If Washington will tell us the competencies desired, we can supply the teaching materials, but as yet no specific manual for instruction is forthcoming. He pointed out errors in some of the War manuals. On page 4 of Mathematics for Pilot Training, he said that five out of eight answers are incorrect. Many of the teaching examples are absurd. Many of the verbal problems are artificial and cannot meet the needs in civil life or the military needs. The graphs violate good graphic technique.

Due to the lack of time, Mr. Schorling did not finish his speech but outlined some of the things that should be stressed in pre-induction mathematics courses—whole numbers and decimals, scale drawings, reading of maps and graphs, ratio and proportion, simple formulas and equations, simple constructions, percentage (3 case), direct and indirect measurements.

Many teachers stayed after the meeting to hear more of his speech and discuss it with him.

The attendance at the afternoon session was about 300.

The fifth annual Mathematics Luncheon was held at twelve o'clock in Room 314 North, Illini Union Building. Miss Gertrude Hendrix, Eastern Illinois State Teachers College, Charleston, Illinois, gave the after dinner talk on "The Classroom Road to Dynamic Knowledge."

A vote of thanks was given Dr. Hartley, Chairman for luncheon arrangements, for the delightful meal and the good speaker. There were sixty present for the luncheon.

Lois Busby, *Secretary*

Teachers College, Columbia University will offer the following courses in the teaching of mathematics this summer, July 6 to August 13.

By Professor John R. Clark: Business mathematics; Teaching algebra in secondary schools. Dr. Nathan Lazar: Teaching geometry in secondary schools. Mr. G. R. Mirick: Elementary mechanics; Observation and participation in the teaching of mathematics. Professor W. D. Reeve: Teaching and supervision of mathematics: senior high school; The teaching of mathematics related to war courses. Professor C. N. Shuster: Mathematics applied to elementary military engineering; Pre-flight training in Civil Aeronautics Navigation; Professionalized subject matter for teachers and students wishing an introduction to navigation. Miss Ethel Sutherland: Teaching arithmetic in primary grades; Teaching arithmetic in intermediate grades.

There will be held during the Summer Session, on consecutive Thursdays beginning July 8, five informal conferences in which all the instructors in mathematics will take part for the purpose of bringing before the students vital questions affecting the present reorganization of mathematical instruction. There will be opportunity for discussion in which the students are invited to participate fully. These conferences serve as a common meeting place for all students and instructors and thus serve both professional and social ends. Registration for these conferences is not necessary.

BOOK REVIEWS

A Source Book of Mathematical Applications. Compiled by a Committee of the National Council of Teachers of Mathematics: Edwin G. Olds, Chairman, Lee E. Boyer, Ruth O. Lane, Nathan Lazar and F. Lynwood Wren. Seventeenth Yearbook of the National Council of Teachers of Mathematics, 1942. New York: The Bureau of Publications, Teachers College, Columbia University. xvi+291 pp. Price, $2.00.

Of the many excellent yearbooks of the National Council of Teachers of Mathematics none has been more timely in its appearance nor of more immediate use to teachers. At a time when mathematics teachers are in need of practical applications of mathematics to vitalize their wartime teaching, this book appears with a wealth of usable material.

Applications of mathematics are given under the headings: arithmetic, algebra, geometry and trigonometry. Under each of those classifications direct vocational applications are given. The authors state that there has been no attempt to make the list of applications for any topic exhaustive. They express the hope that the list of applications will be suggestive. A brief summary of a few of the applications indicates that the authors have been modest in their statement.

While applications from such fields as shop, mechanics, physics, engineering, electricity, pattern making, measurement and surveying really have important wartime uses, it is interesting to note the many topics with direct wartime uses. One notices a wealth of such applications under such headings as *aviation:* gasoline supply; map reading; identification letters; airspeed and ground speed; load ratio; Venturi area; propellor speed; air-cooled vs. liquid-cooled engines; oil supply; wind direction; drift angle and correction angle; interception; radius of action; lift and drag; airplane ceiling; horsepower; air pressure; and path of a bomb; *ballistics:* capacity of a shell; law of falling objects; velocity of a shell; *communications: radio* and many others.

While wartime uses of mathematics call for emphasis at this time, the uses of mathematics range from accident data to applications in music; from uses in chemistry to those in the kitchen; from economy in buring vitamin capsules to the amount of snow that falls on a town; from dietetics to rapid calculation; from art to the uses employed by a carpenter.

No teacher of mathematics can affort not to own this book; the progressive teacher will use it many times each week.

Virgil S. Mallory

A First Course in Algebra. By Virgil S. Mallory. Sanborn. 1942. viii+510 pp. Price, $1.48.

This book is a new edition of the Stone-Mallory Algebra that will be of interest to teachers of elementary algebra. In addition to covering the regular course in algebra, this new book has as special features:

1. Four diagnostic tests in arithmetic, eight achievement tests in arithmetic, and a development of and practice in the fundamentals of arithmetic.
2. Many wartime applications of mathematics including simple problems in aeronautics, ballistics, in the application of formulas in simple engineering and shop problems, and in the laws of the lever with applications to problems in physics.
3. A complete review and testing program at the end of each chapter.
4. An emphasis on self teaching devices.

The work on the four big ideas which the well educated citizen should know, namely, the formula, the equation, the graph, and directed numbers, are well treated in this book. Ample exercise material is included to satisfy all needs.

The book is attractively made and bound. Any pupil who masters its contents will be well fitted to continue his work in mathematics and science.

W.D.R.

Basic Mathematics. By William Betz. Ginn. 1942. x+502 pp. Price, $1.48.

This new book is an emergency course covering the basic elements of arithmetic, informal geometry, elementary algebra, and numerical trigonometry. The material is well organized, simple, and well illustrated throughout. Boys and girls who study this book will obtain the kind of training which the armed forces of the country consider essential for pre-induction training in mathematics. Some applications to the problems of aviation will add interest to the work. The greatest value of the book, however, is that it can be taught in a year's time and actually represents the fundamental ideas that all boys and girls should possess.

W.D.R.

Basic Mathematics. By W. W. Hart. Heath. 1942. vi+456 pp. Price, $1.52.

This new book is a survey or refresher course in the fundamental ideas of the mathematics of the secondary school together with applications of the various parts to marine and aerial navigation, to ballistics, mechanics, engineering and science.

The book is so planned that it can be used as a straight survey course of one year as preparation for induction into the armed services or for special training in college.

It contains chapters on arithmetic computation, elementary geometry, mensuration, elementary algebra, logarithms, elementary trigonometry, demonstrative geometry, social geometry and advanced algebra.

Teachers of mathematics who are looking for a book that gives a complete survey of fundamentals together with a wealth of applications will want to see this book.

W.D.R.

Trigonometry, Plane and Spherical, By Miles C. Hartley. vi+298 pp. The Odyssey Press, New York. 1942. Price, $1.60.

This text has been made for use in either high schools or colleges and is so arranged that courses of various lengths may be given. The preface states that the "text has been developed with the firm belief that trigonometry can be made, and should be made, more meaningful. . . . Special attention has been given to the problem of simplifying the subject for the beginning student."

Examination of the text shows that the author, with many teaching devices, has succeeded in this attempt. The book begins with a brief review of algebra and geometry and has an early introduction to logarithms, thus providing continued practice in their use throughout the course.

Explanation of significant digits and of the accuracy of a computed result gives a sensible approach to the use of the four-place table of logarithms. The section on spherical trigonometry will be welcomed by those schools teaching such a course as a war measure.

VIRGIL S. MALLORY